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## ГІДРОТЕХНІЧНЕ БУДІВНИЦТВО

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**Dovbenko I. E.** [1; ORCID ID: 0000-0001-8393-1887],
PhD student
**Voloshchuk V. A.** [2; ORCID ID: 0000-0003-0687-8958],
Doctor of Technical Sciences, Head of Department
[1] *National University of Water and Environmental Engineering, Rivne*
[2] *National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"*

# INVESTIGATION OF THE FRICTION FACTOR IN HYDRAULICALLY ROUGH PIPES

**The study presents the results of a comprehensive theoretical and analytical investigation of the friction factor in pipes with a uniformly rough internal surface. Based on the principles of dimensional analysis, a detailed mathematical model was developed to determine the friction factor within the pre-quadratic zone of turbulence, where the flow regime transitions between smooth and fully rough conditions. The proposed model accounts for the influence of relative roughness, Reynolds number, and flow structure parameters, enabling a more accurate description of hydraulic resistance under turbulent flow. To validate the theoretical findings, the classical experimental data obtained by J. Nikuradse for pipes with uniform roughness were subjected to modern statistical processing and comparative evaluation. Furthermore, a conceptual physical model was introduced to describe the dynamic interaction of vortices in the boundary layer formed above the rough pipe surface. The results contribute to a deeper understanding of turbulence mechanisms and provide a basis for improving the prediction of energy losses in hydraulic systems with rough-walled conduits.**

**Keywords:** theoretical research, friction factor, hydraulically rough pipes, region of pre-quadratic turbulence, dimensional analysis method.

**1. Introduction.** Challenges associated with the hydraulic computation of pipelines often arise when solving hydrodynamic problems. The motion of a liquid through a closed conduit occurs either because of differences in geodetic elevation or as a result of energy imparted to the flow by pumping equipment. In engineering practice, hydraulic calculations are generally performed to determine head

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losses and to identify the most economical pipe diameter corresponding to a given discharge rate [1-3].

In 1752, Leonhard Euler formulated the Bernoulli equation for the motion of a real fluid, which expresses the energy balance between two cross-sections of a flow:

$$z_1 + \frac{p_1}{\rho g} + \frac{u_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + h_w, \quad (1)$$

where $z_1$ and $z_2$ - represent the elevations of the flow above a reference plane (m); $p_1$ and $p_2$ - pressure values (Pa); $u_1$ and $u_2$ – flow velocities (m/s); $\rho$ – fluid density (kg/m3); $g$ - gravitational acceleration, $g = 9.81$ (m/s2) and $h_w$ denotes the head loss due to friction along the pipe length.

Later, in 1845, Julius Weisbach and, in 1857, Henri Darcy proposed an empirical relationship to calculate head losses along a pipe (Colebrook & White 1937; Kuznietsov 2022; Guo 2002):

$$h_w = \lambda \frac{l}{d} \frac{\bar{u}^2}{2g}, \quad (2)$$

where $\lambda$ is the friction factor, $\bar{u}$ is the mean flow velocity (m/s), $l$ is the pipe length (m), and $d$ is the pipe diameter (m).

This expression is known as the Darcy–Weisbach equation. The friction factor $\lambda$ depends on the pipe geometry, internal surface roughness, and the flow regime.

Different types of flow correspond to different patterns of fluid particle motion. These flow types were first experimentally studied by Osborne Reynolds, who established the existence of two primary flow regimes. At low velocities, fluid motion is orderly and stratified referred to as laminar flow. At higher velocities, the motion becomes chaotic and dominated by small-scale eddies known as turbulent flow. Reynolds' classical experiments (1868) and subsequent research demonstrated that a gradual increase in flow velocity preserves laminar motion up to a certain critical value, beyond which the flow becomes turbulent. Conversely, when velocity decreases, turbulent motion persists down to another critical threshold before returning to laminar conditions [4-8].

Reynolds introduced the concept of critical velocity, which marks the transition between laminar and turbulent regimes. Two critical velocities exist: the lower $\bar{u}_{l.v}$, where turbulence decays into laminar flow, and the upper $\bar{u}_{u.v}$ , where laminar flow transitions into turbulence, with the inequality $\bar{u}_{l.v} < \bar{u}_{u.v}$ always satisfied.

The dependence between head loss $h_w$ and mean velocity $\bar{u}$ differs for laminar and turbulent flows. Figure 1 presents the logarithmic relationship between these parameters.

In laminar conditions, when $\bar{u}_{l.v}$ (point a), head losses vary proportionally to the first power of velocity, while for turbulent flow $\bar{u}_{u.v}$ (point b), losses vary approximately with 1.75–2.0. The intermediate range between points a and b represents the transition zone between laminar and turbulent motion.

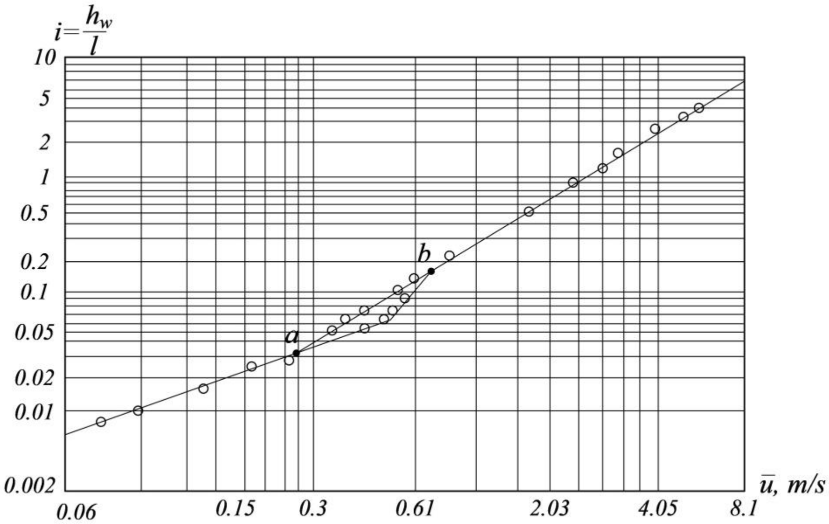


Fig. 1. Relationship between head loss and mean velocity for different flow regimes

Subsequent research established that flow type depends not only on velocity but also on fluid viscosity and the geometry of the conduit. Hence, the flow regime is characterized by a dimensionless parameter - the Reynolds number:

$$Re = \frac{\bar{u}d}{\nu}, \quad (3)$$

where $d$ is the pipe diameter (m) and ; $\nu$ is the kinematic viscosity of the fluid (m²/s).

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In engineering applications, the lower critical Reynolds number ($Re_{cr}$) is commonly used to distinguish between laminar and turbulent regimes.

J. Nikuradse (1932, 1933) performed pioneering investigations on the relationship between the Reynolds number and the friction factor for both hydraulically smooth and uniformly rough pipes. His experiments produced a series of characteristic curves (Fig. 2) showing the dependence of the friction factor $\lambda$ on $Re$ and relative roughness.

In laminar flow ($Re \leq 2320$), the fluid moves smoothly and bypasses surface irregularities, making the effect of roughness negligible. Thus, local frictional resistance is minimal. On the friction factor diagram (Fig. 2), this corresponds to a straight line, and the values can be calculated directly using the Darcy–Weisbach relationship.

In contrast, in turbulent motion, the higher flow velocity produces steep velocity gradients between adjacent fluid layers, leading to the generation of vortices within the boundary layer. These complex interactions cause increased resistance and energy dissipation [3].

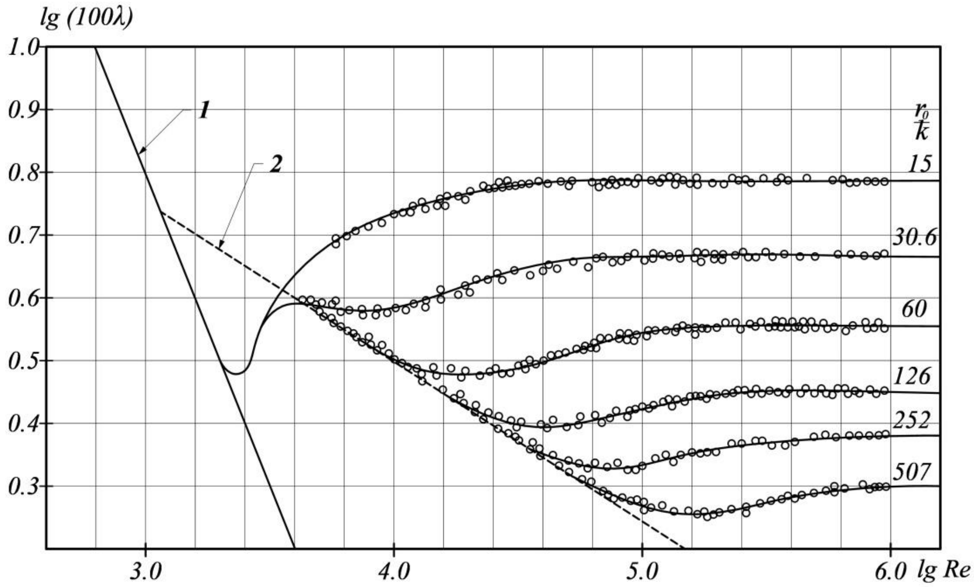


Fig. 2. Dependence of friction factor on Reynolds number and pipe surface roughness (after J. Nikuradse): 1 – laminar flow; 2 – hydraulically smooth turbulent region (G. Blasius).

Numerous researchers have conducted both theoretical and experimental studies to derive analytical or empirical expressions for estimating the friction factor as a function of the main flow and geometric parameters.

**2. Methods and Techniques.** The research methodology combines theoretical analysis, dimensional modeling, and statistical data interpretation. Analytical relationships were established between the Reynolds number, relative roughness, and the friction coefficient within the pre-quadratic turbulence zone. To verify the theoretical model, the classical experimental results obtained by J. Nikuradse were digitized and reprocessed using regression and correlation analysis methods. This enabled the evaluation of the model's predictive accuracy and its consistency with empirical trends. Additionally, a conceptual vortex interaction model was formulated to describe energy exchange processes within the boundary layer. The combination of analytical and statistical techniques ensured both theoretical validity and practical applicability of the obtained results.

The present study focuses on developing a general mathematical framework for determining the friction factor of turbulent flow in pipes, as expressed in Equation (2).

**3. Results and Discussion.** To derive a universal approach for quantitatively assessing head losses during the motion of a real fluid, it is necessary to identify how wall shear stresses depend on the principal physical and geometric parameters of the system. The dominant factors influencing friction along the inner surface of a pipe include: fluid density $\rho$, dynamic viscosity $\mu$, hydraulic radius $R$, average roughness height $k$, and mean flow velocity $\bar{u}$.

Using the dimensional analysis method, the general functional dependence of the wall shear stress on these parameters can be expressed as:

$$\tau_0 = k\rho^z \mu^m R^p \varepsilon^x \bar{u}^n \quad (4)$$

which, in dimensional terms, may be represented by:

$$\frac{Ml}{t^2}\frac{1}{l^2} = \frac{M^z}{l^{3z}}\frac{M^m}{l^m t^m} l^p l_i^x \frac{l^n}{t^n}, \quad (5)$$

where $M$, $l$, and $t$ denote mass, length, and time dimensions, respectively.

By equating the dimensional exponents for both sides of Equation (5), we obtain the following relationships:

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$$\left.\begin{aligned} for\,the\,mass:1&=m+z \\ for\,length:-1&=n+p-m-3z+x \\ for\,time:-2&=-n-m \end{aligned}\right\} . \quad (6)$$

Since the parameters $n$ and $x$ are known, additional degree-related indicators can be determined from system (6)

$$\left.\begin{aligned} m&=2-n \\ z&=n-1 \\ p&=n-2-x \end{aligned}\right\} . \quad (7)$$

By substituting the obtained degree indicators $m$, $z$, and $p$ in terms of $n$ and $x$ into equation (4), the expression for the tangential stresses on the inner surface of the pipe is derived

$$\tau_0 = K\rho^{n-1}\mu^{2-n}R^{n-2-x}k^x\overline{u}^n = K\rho^{n-1}\rho^{2-n}\nu^{2-n}\overline{u}^nR^{n-2}\frac{k^x}{R^x} . \quad (8)$$

$$\tau_0 = K\rho\frac{R^{n-2}\overline{u}^{n-2}\overline{u}^2}{\nu^{n-2}}\frac{k^x}{R^x} = K\rho\left(\frac{k}{R}\right)^x\frac{\overline{u}^2}{Re^{2-n}} . \quad (9)$$

$$\tau_0 = K\rho\left(\frac{k}{R}\right)^x\frac{\overline{u}^2}{Re^{2-n}} . \quad (10)$$

The tangential stresses acting on the pipe's inner surface can be expressed by the following equation

$$\tau_0 = \frac{\lambda}{8}\rho\overline{u}^2 . \quad (11)$$

The friction factor $\lambda$ is determined by equating the right-hand sides of equations (10) and (11)

$$\frac{\lambda}{8}\rho\overline{u}^2 = K\rho\left(\frac{k}{R}\right)^x\frac{\overline{u}^2}{Re^{2-n}} . \quad (12)$$

$$\lambda = \frac{8K}{Re^{2-n}}\left(\frac{k}{R}\right)^x = \frac{8K}{Re^{2-n}}\left(\frac{4k}{d}\right)^x = \frac{8K}{Re^{2-n}}\left(\frac{2k}{r_0}\right)^x . \quad (13)$$

$$\lambda = \frac{8K}{Re^{2-n}}\left(\frac{2k}{r_0}\right)^x . \quad (14)$$

The indicator of degree $x$ takes into account the effect of the roughness of the inner surface of the pipe $k/r_0$ on the friction factor, and the indicator of degree $n$ - the effect on the friction factor of the flow regime. Experimental studies of scientists prove that in laminar flow $x=0$, $n=1$, and in turbulent flow in the region of quadratic turbulence $x=1$, $n=2$.

Then in laminar flow, when $x=0$, $n=1$ and $K=8$, equation (14) will have the form

$$\lambda_{lam}=\frac{8K}{Re}=\frac{64}{Re}, \quad (15)$$

and in turbulent flow in the region of quadratic turbulence when $x=1$, $n=2$ – to the form

$$\lambda_{q\,t}=16K\frac{k}{r_0}=const. \quad (16)$$

In the region of quadratic turbulence, the friction factor does not change and is equal to the characteristic of the roughness of the inner surface of the pipe.

In the article [3], L. Prandtl presented the dependence of the friction factor $\lambda_{q\,t}$ on the relative roughness of the inner surface of the pipe $r_0/k$

$$\lambda_{q\,t}=\frac{1}{\left(1.74+2\log\frac{r_0}{k}\right)^2}. \quad (17)$$

For a turbulent flow, in the region of hydraulically smooth turbulence, J. Blasius, based on experimental studies, taking $x=1$, $n=1.75$ and $K=0.0395$, obtained the following equation

$$\lambda_{s\,t}=\frac{0.3164}{Re^{0.25}}. \quad (18)$$

This equation is adequate to experimental data for Reynolds numbers $Re<80000$. At higher Reynolds numbers, it reduces the value of the friction factor.

The author proposed the following mathematical model for the region of hydraulically smooth turbulence [3].

$$\lambda_{s\,t}=64\left(\frac{0.01034}{Re^{0,5}}+\frac{0.003124}{Re^{0,25}}+0.0000726\right). \quad (19)$$

As shown in the named article, this mathematical model is adequate to experimental data within the entire region of hydraulically smooth turbulence.

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It is known [5] that in practice the flow in pipelines moves in a turbulent regime, mostly in the region of pre-quadratic turbulence. But currently, for pipes with uniform roughness, there is no calculated dependence of the friction factor for this region. Visually analyzing the graphs that characterize the dependence of the friction factor on the Reynolds number within the region of pre-quadratic turbulence (Fig. 2), it is obvious that there is a geometric similarity between them. This can be explained by the fact that the technology of creating the inner surface in these pipes was the same. In each case, it was created by covering it with sand of the accepted diameter [3].

As a characteristic of the roughness of the inner surface of the pipes, J. Nikuradse adopted the ratio of geometric quantities $r_0/k$. It can be seen from Figure 2 that the ratio $r_0/k$ and the Reynolds number $Re$ uniquely determine the shape and position of the graphs $lg(100\lambda) = f(r_0/k, lg\,Re)$. At the same time, it should be noted that J. Nikuradse performed the research on pipes of various diameter and with different average heights of roughness $k$.

The purpose of this work is to obtain a mathematical model for calculating the friction factor in the region of pre-quadratic turbulence in pipes with uniform roughness.

We have accepted the hypothesis that within the region of pre-quadratic turbulence, the degree $n$ indicator is variable and takes the following fixed intermediate values: $n=1.25$, $n=1.5$, and $n=1.75$. Then, the mathematical model for determining the value of the friction factor for pipes in the region of pre-quadratic turbulence, taking into account equation (14) and fixed values of the exponent of the power of $n$, can be represented by the sum of terms

$$\lambda_{pq\,t} = \frac{k}{r_0}\left(\frac{K_1}{Re^{0.75}} - \frac{K_2}{Re^{0.5}} + \frac{K_3}{Re^{0.25}}\right). \quad (20)$$

Each term of the mathematical model (20) has an unknown coefficient of $K_i$. These coefficients are determined by the method of least squares on the basis of the statistical processing of experimental data.

Figure 3 shows experimental points from the experimental data of J. Nikuradse for the region of quadratic turbulence. On the left, they are bounded by a graph that corresponds to the region of hydraulically smooth turbulence. On the right, in the region of quadratic turbulence, are continued by the graph (horizontal lines) according to L. Prandtl's equation.

Graphs according to the mathematical model (20) are given within the limits of the quadratic turbulence.

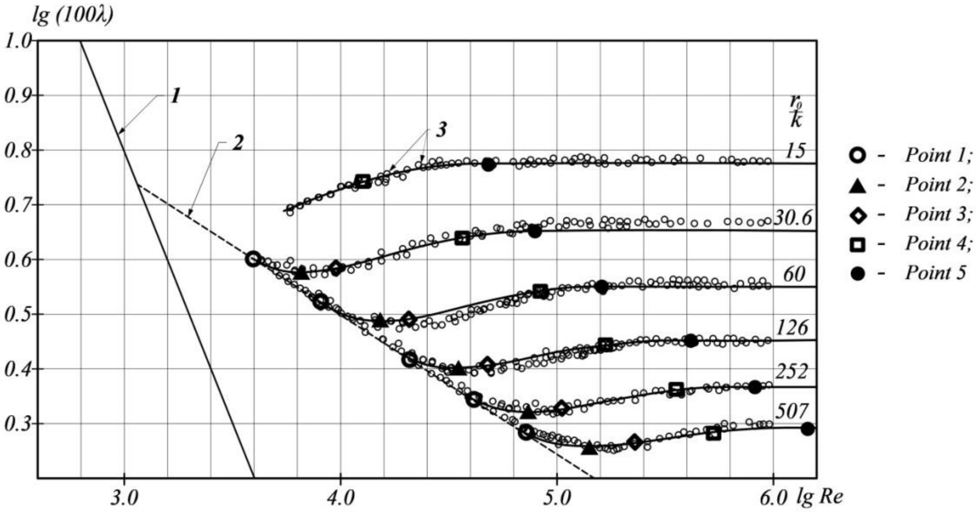


Fig. 3. Graphs of the dependence of the friction factor on the Reynolds number and the roughness of the inner surface of the pipes: 1 – laminar flow; 2 – region of hydraulically smooth turbulence (G. Blasius); 3 – research experimental points (according to J. Nikuradse)

Table 1 shows pipe diameters $d$, average roughness heights $k$, Reynolds numbers corresponding to the limit of hydraulically smooth and pre-quadratic turbulence $lg\,Re_{s\,t}$ and the limit of pre-quadratic and quadratic turbulence $lg\,Re_{q\,t}$, the roughness of the inner surface of the pipes $r_0/k$, the friction factor $\lambda_{q\,t}$, as well as the numerical values of the coefficients $K_i$, degrees of freedom and tabular and calculated values of Fisher's criteria, relative to the mathematical model (20).

As can be seen from Table 1, the calculated Fisher criteria are in all cases smaller than the tabular Fisher criteria, which confirms with a reliability of 95% the adequacy of the mathematical model of the authors (20) to the experimental data in the field of quadratic turbulence, which were obtained on the basis of hydraulic studies by J. Nikuradse.

All graphs $lg(100\lambda) = f(r_0/k, lg\,Re)$ in the area of pre-quadratic turbulence, except for the graph with pipe roughness $r_0/k = 15$, have a

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similar shape and consist of two sections. The first section has a concave shape, and the second - has a convex shape. The beginning of each graph is a certain flow regime, in which its transition from hydraulically smooth turbulence to pre-quadratic turbulence occurs. This state is determined by the Reynolds number $Re_{s\,t}$, at which the values of the friction factor, which are determined by equations (18) and (20), respectively, are equal to each other $\lambda_{s\,t} = \lambda_{pq\,t}$.

Table 1.

Parameters of hydraulic research of pipes with uniform roughness performed by J. Nikuradse, and the results of their statistical analysis

| $\frac{d}{k}$ | $Re_{s\,t}$ | $Re_{q\,t}$ | $\frac{r_0/k}{\lambda_{q\,t}}$ | *The value of the coefficients* | | | $\frac{f_a}{f_в}$ | $\frac{F_m}{F_p}$ |
|---|---|---|---|---|---|---|---|---|
| | | | | $K_1$ | $K_2$ | $K_3$ | | |
| $\frac{2.412}{0.08}$<br>$\frac{4.820}{0.16}$ | *2405* | *47978* | $\frac{15}{0.0667}$ | *112.3* | *347.1* | *31.6* | $\frac{19}{21}$ | $\frac{2.14}{1.11}$ |
| $\frac{2.434}{0.04}$<br>$\frac{4.870}{0.08}$<br>$\frac{9.640}{0.16}$ | *3928* | *78364* | $\frac{30{,}6}{0.0327}$ | *2960.0* | *751.9* | *57.4* | $\frac{27}{29}$ | $\frac{1.87}{1.07}$ |
| $\frac{2.434}{0.02}$<br>$\frac{9.800}{0.08}$ | *7998* | *159590* | $\frac{60}{0.0167}$ | *8289.7* | *1718.1* | *108.0* | $\frac{25}{27}$ | $\frac{1.94}{1.08}$ |
| $\frac{2.474}{0.01}$<br>$\frac{9.920}{0.04}$ | *20741* | *413835* | $\frac{126}{0.00794}$ | *23758.0* | *4071.0* | *214.1* | $\frac{32}{34}$ | $\frac{1.80}{1.06}$ |
| $\frac{4.940}{0.01}$<br>$\frac{9.940}{0.02}$ | *41075* | *819553* | $\frac{252}{0.00397}$ | *66163.0* | *9478.1* | *419.1* | $\frac{25}{27}$ | $\frac{1.94}{1.08}$ |
| $\frac{9.940}{0.01}$ | *72110* | *1438792* | $\frac{507}{0.00197}$ | *167672* | *20767* | *803.3* | $\frac{28}{30}$ | $\frac{1.87}{1.07}$ |

In the first section, as the Reynolds number increases, the graph decreases monotonically and the coefficient $\lambda$ reaches its extreme minimum value at the point where

$$\frac{d\lambda_{pq\,t}}{d\,Re}=\frac{d}{d\,Re}\left(\frac{k}{r_0}\left(\frac{K_1}{Re^{0.75}}-\frac{K_2}{Re^{0.5}}+\frac{K_3}{Re^{0.25}}\right)\right)=0 \quad . \qquad (21)$$

Taking the derivative of equation (21), get the equation

$$\frac{d\lambda_{pq\,t}}{d\,Re}=0.75K_1\,Re^{0.25}-0.5K_1\,Re^{0.5}+0.25K_1\,Re^{0.75}=0 \quad . \qquad (22)$$

The root of this equation is the Reynolds number $Re$, which describes the flow regime at which the friction factor $\lambda_{pq\,t}$ reaches its extreme minimum value.

As the Reynolds number increases further, the graph grows and reaches an inflection point at which the concave graph turns into a convex graph. At this point, the condition is fulfilled

$$\frac{d^2\lambda_{pq\,t}}{d\,Re^2}=\frac{d^2}{d\,Re^2}\left(\frac{k}{r_0}\left(\frac{K_1}{Re^{0.75}}-\frac{K_2}{Re^{0.5}}+\frac{K_3}{Re^{0.25}}\right)\right)=0 \quad . \qquad (23)$$

Taking the second derivative of equation (21)

$$\frac{d^2\lambda_{pq\,t}}{d\,Re^2}=0.75\times0.25K_1\,Re^{-0.75}-0.5\times0.5K_1\,Re^{-0.5}+0.25\times0.75K_1\,Re^{-0.25}=0 \quad . \qquad (24)$$

The root of this equation is a Reynolds number of $Re$, which corresponds to the inflection point at which a concave graph becomes a convex graph.

With a further increase in the Reynolds number, the graph grows and reaches its maximum position at a point with a Reynolds number of $Re_{q\,t}$, which corresponds to the transition of the flow regime from pre-quadratic turbulence to quadratic turbulence. The friction factor $\lambda_{q\,t}$ at this state of the flow is determined depending on the roughness of the inner surface of the pipe $r_0/k$ according to L. Prandtl's equation (17).

In the mathematical model (20), the first and second terms take into account the influence of the roughness of the friction factor on the first section of the graph, where it is concave. The third term takes into account the influence of the roughness of the flow in the second section, where the graph is convex. Together, all terms of model (20) take into account the shape and position of the graph.

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Figure 4 shows graphs of the dependence of coefficients $K_1$ and $K_2$ on the roughness of the inner surface of pipe $r_0/k$ . It can be seen from the graph that with increased roughness of the inner surface of the pipe, these coefficients increase parabolically.

Figure 5 shows a graph of the dependence of coefficient $K_3$ on the roughness of the inner surface of pipe $r_0/k$ . The graph shows that with increased roughness of the inner surface of the pipe, the coefficient increases linearly.

The change of coefficients $K_1$ and $K_2$ parabolically and coefficient $K_3$ linearly from the roughness of the inner surface of the pipe confirms the accepted hypothesis that graphs $lg(100\lambda) = f(r_0/k, lg\,Re)$ are similar to each other and are described by the mathematical model proposed by the authors (20).

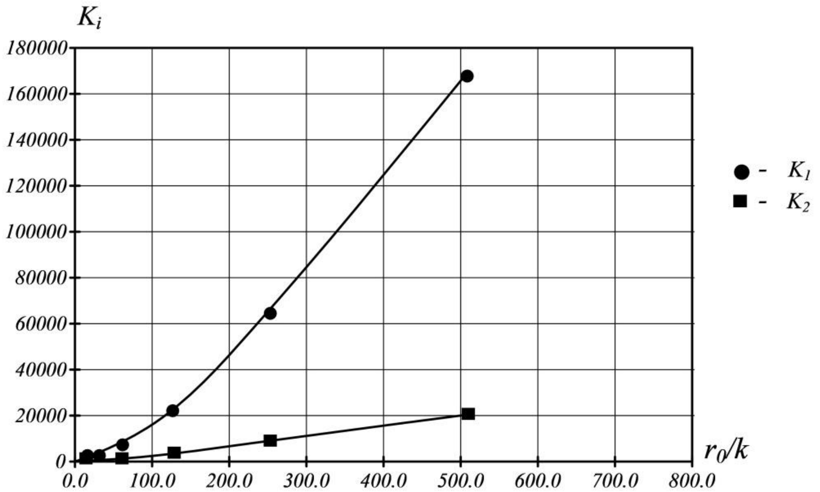


Fig. 4. Graphs of the dependence of coefficients $K_1$ and $K_2$ on the roughness of the inner surface of the pipe

The article [6] proved that the boundary layer existing on the perimeter of the inner surface of the pipe is a set of ring vortices. The vortex axis of these vortices coincides with the cross-section of the flow, and their transverse diameter depends on the Reynolds number (Fig. 6). The diameter of the vortex is equal to the thickness of the boundary laminar layer and is determined by the equation

$$d_{vor} = \delta = \frac{Nd}{\text{Re}}\sqrt{\frac{\lambda_{pq\,t}}{8}}\,, \qquad (25)$$

where $N$ - the Nikuradse number, which is equal to 10.5–11.0; $d$ – pipe diameter, m.

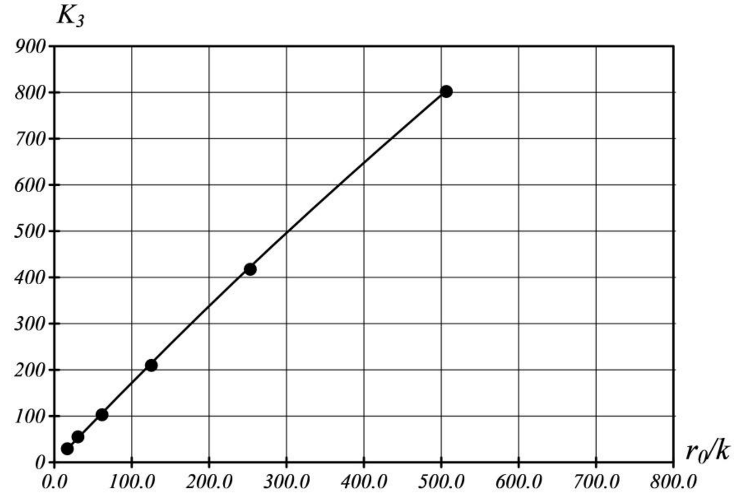


Fig. 5. Graph of the dependence of coefficient $K_3$ on the roughness of the inner surface of the pipe

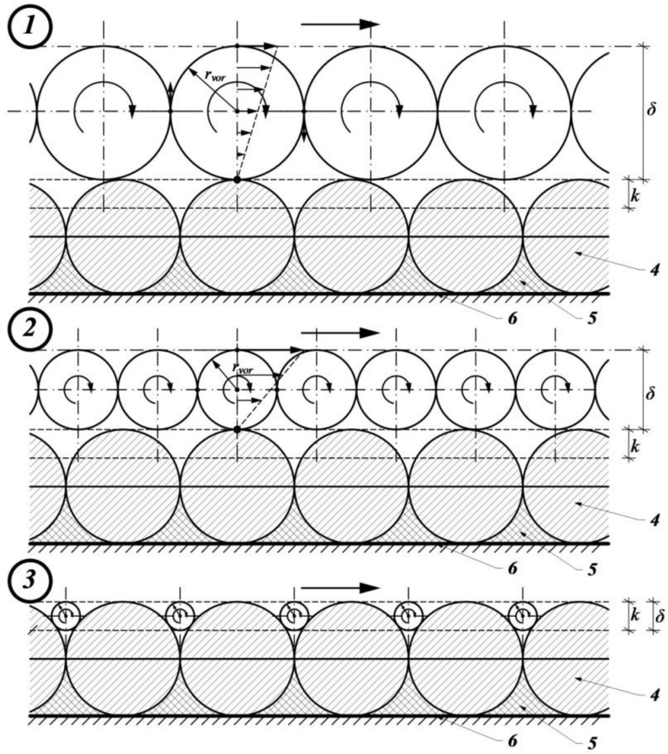


Fig. 6. Scheme of the interaction model in the vortex boundary layer with uniform roughness of the inner surface of the pipe wall: 1 –

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area of hydraulically smooth turbulence; 2 – mode of incomplete manifestation of roughness; 3 – mode of full display of roughness; 4 – grains of uniform roughness; 5 – glue (varnish); 6 – pipe wall; $k$ - roughness height, m; $\delta$ - boundary layer thickness or vortex diameter, m.

As the Reynolds number increases, the diameter of the vortex decreases accordingly. With increased roughness, namely, friction factor $\lambda_{pqt}$ - increases.

The uniqueness of J. Nikuradse's experiments lies in the fact that they make it possible to compare the diameter of the vortex $d_{vor}$ and the height of the roughness $k$.

Reduce the dependence (25) to the form

$$\frac{r_0}{r_{vor}} = \frac{d}{d_{vor}} = \frac{Re}{N}\sqrt{\frac{8}{\lambda}}, \quad (26)$$

Figure 7 shows the relationships between the radius of the vortex and the height of the roughness at characteristic points depending on the roughness of the pipe.

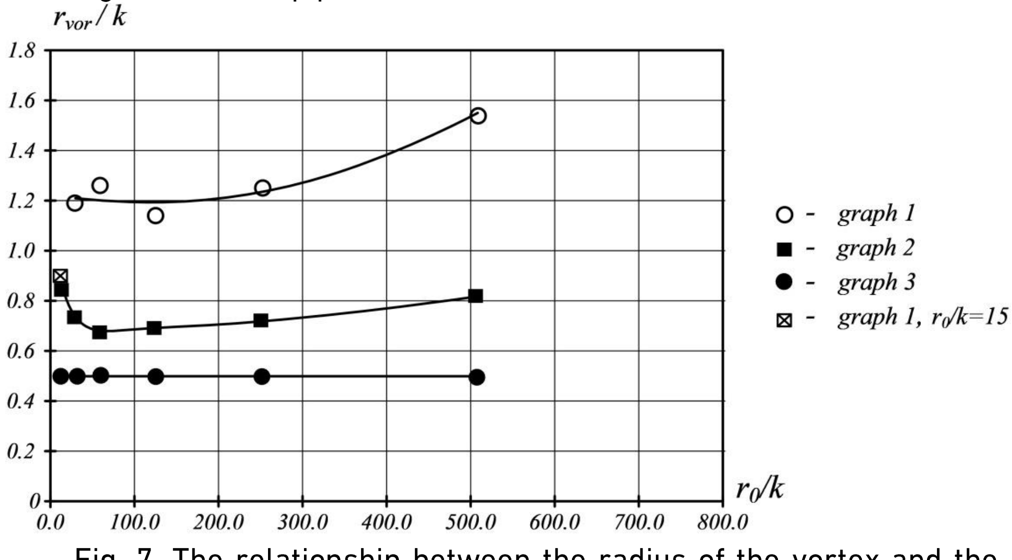


Fig. 7. The relationship between the radius of the vortex and the height of roughness at characteristic points depending on the roughness of the pipe

In Figure 7, graph 1 describes the flow regime in which its transition from hydraulically smooth turbulence to pre-quadratic turbulence occurs. It can be seen from the graph that in this case the ratio of the radius of the vortex to the height of the roughness is greater than one. That is, the thickness of the boundary layer (created by the vortex) in all cases, except for the case of $r_0/k = 15$, exceeds the height of the roughness by more than 2 times. Such conditions provide a regime of hydraulically smooth turbulence.

Graph 2 describes the flow regime in which friction factor $\lambda_{pq\,t}$ reaches its extreme minimum value. It can be seen from the graph that in this case, the ratio of the radius of the vortex to the height of the roughness is greater than $r_{vor}/k > 0.5$. Such conditions ensure the regime of incomplete roughness.

Graph 3 describes the flow regime in which the ratio of the vortex radius to the roughness height is $r_{vor}/k = 0.5$. Such conditions ensure the full roughness regime. In this case, the vortices of the boundary layer are completely located within the roughness level. It should be noted that with a further increase in the Reynolds number, the diameter of the boundary layer vortex does not decrease.

Figure 8 shows the graphs of the dependence of the Reynolds number on the roughness of the inner surface of the pipe for typical flow regimes.

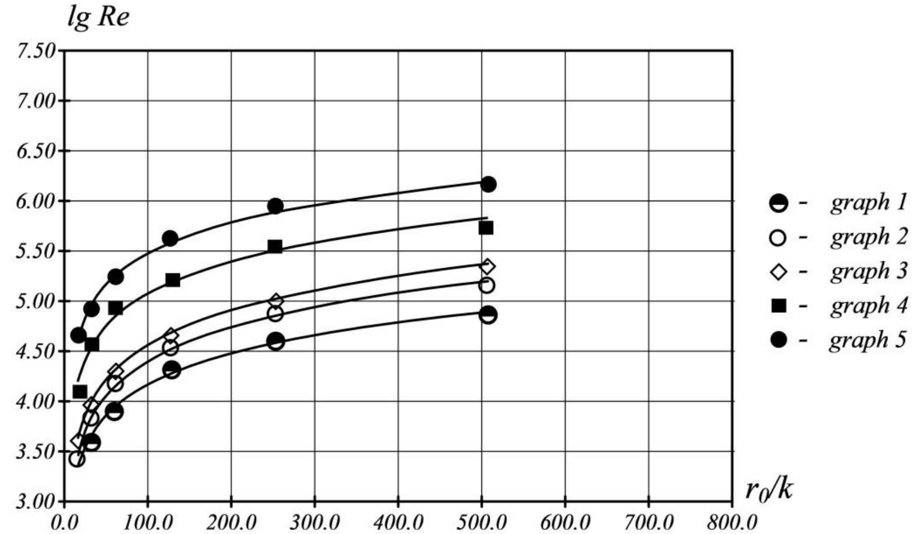


Fig. 8. Graphs of the dependence of the Reynolds number on the roughness of the inner surface of the pipe for characteristic flow regimes: 1 – the flow regime in which its transition from hydraulically smooth turbulence to pre-quadratic turbulence occurs; 2 – the flow

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regime in which the coefficient $\lambda_{pqt}$ reaches its extreme minimum value; 3 – the state of the flow, in which the ratio of the vortex radius to the roughness height is $r_{vor}/k = 0.5$; 4 – the flow regime corresponding to the inflection point at which the concave graph turns into a convex one; 5 – the mode of flow transition from pre-quadratic turbulence to quadratic turbulence

Using Figure 8, it is possible to determine the Reynolds number for characteristic points that describe the considered flow regime, depending on the roughness of the inner surface of the pipe.

4. Conclusions

1. To develop a general method of quantitative accounting of pressure losses in a real fluid flow based on the dimensional analysis method, a mathematical equation was established for the dependence of the friction forces on the inner surface of the pipe on the main operating factors, namely the roughness of the inner surface of the pipe $k/r_0$ and the turbulence of the flow, which is determined by the Reynolds number.

2. The accepted hypothesis is that the mathematical model for determining the value of the friction factor for pipes in the region of pre-quadratic turbulence, taking into account the internal roughness of the pipe and fixed values of the power $n$, can be represented by the sum of the corresponding terms.

3. Based on the experimental data of I. Nikuradse, the adequacy check of the obtained mathematical model was performed.

4. It has been established that in the mathematical model for the region of hydraulically smooth turbulence, the degree of flow turbulence is taken into account by indicators of degrees $n = 1.5$, $n = 1.75$ and $n = 2.0$, and for the region of pre-quadratic turbulence - by indicators of degrees $n = 1.25$, $n = 1.5$ and $n = 1.75$.

5. A mathematical analysis of the shape of dependence graph $lg(100\lambda) = f(r_0/k, lg\,Re)$ was performed. It was established that all graphs are geometrically similar to each other and have concave and convex sections. As the roughness increases, the graph moves to the left and up.

6. The paper presents graphs of the dependence of coefficients $K_i$ on the roughness of the inner surface of the pipe $r_0/k$. It was

established that with increased roughness of the inner surface of the pipe, these coefficients increase according to the corresponding laws.

7. For the first time, the values of the parameters of the characteristic points on the graphs were mathematically determined.

8. The scheme of the model of interaction in the boundary layer of the vortex with uniform roughness of the inner surface of the pipe is developed. The ratio between the dimensions of the roughness and the diameter of the vortices was determined for the characteristic points.

9. In the following articles, it will be shown how the turbulence of the flow in steel and cast iron pipes will be taken into account in the mathematical model for the region of quadratic turbulence.

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**Довбенко І.Е.** [1; ORCID ID: 0000-0001-8393-1887],
аспірант
**Волощук В.А.** [2; ORCID ID: 0000-0003-0687-8958],,
Доктор технічний наук, завідувач кафедри
*[1] Національний університет водного господарства та природокористування, Рівне*
*[2] Національй технічний університет Укра\ни «Київській політехнічний інститут імені Ігоря Сікорського"*

## ДОСЛІДЖЕННЯ КОЕФІЦІЄНТА ТЕРТЯ В ГІДРАВЛІЧНО ШОРСТКІЙ ТРУБІ

**У дослідженні представлені результати комплексного теоретичного та аналітичного дослідження коефіцієнта тертя в трубах з рівномірно шорсткою внутрішньою поверхнею. На основі принципів розмірного аналізу було розроблено детальну математичну модель для визначення коефіцієнта тертя в межах передквадратичної зони турбулентності, де режим потоку переходить між гладким та повністю шорстким станами. Запропонована модель враховує вплив відносної шорсткості, числа**

**Рейнольдса та параметрів структури потоку, що дозволяє точніше описувати гідравлічний опір турбулентному потоку. Для перевірки теоретичних висновків класичні експериментальні дані, отримані Дж. Нікурадсе для труб з рівномірною шорсткістю, були піддані сучасній статистичній обробці та порівняльній оцінці. Крім того, було введено концептуальну фізичну модель для опису динамічної взаємодії вихорів у прикордонному шарі, що утворюється над шорсткою поверхнею труби. Результати сприяють глибшому розумінню механізмів турбулентності та забезпечують основу для покращення прогнозування втрат енергії в гідравлічних системах з трубопроводами з шорсткою поверхнею.**

**Ключові слова:** теоретичне дослідження, коефіцієнт тертя, гідравлічно шорсткі труби, область передквадратичної турбулентності, метод розмірного аналізу.

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