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METHODOLOGY FOR OBTAINING SMOOTHED EQUIVALENT LOADING FROM IMPULSE SIMULATION DATA

This paper presents a mathematical methodology for transforming a discrete set of impulse events, obtained from numerical simulations or field observations, into a continuous smooth loading function suitable for subsequent use in finite element analysis, modal analysis, and structural design verification. The need for such transformation arises in problems of blast fragment–structure interaction, multi-body collisions, and contact impact processes, where the loading data takes the form of discrete impulse–time pairs rather than a smooth pressure profile. The methodology is based on a mathematical filter constructed from the equation of motion of a single-degree-of-freedom (SDOF) system at critical damping (damping ratio $\xi = 1$). We derive an analytical expression for the filter kernel — the impulse response function $h(t) = \omega_n^2 \cdot t \cdot \exp(-\omega_n \cdot t)$ — which satisfies the impulse preservation condition $\int h(t) dt = 1$. We provide both physical (conservation of momentum) and mathematical (convolution theorem in the frequency domain) justifications for the unit normalization of the kernel. We justify the use of linear convolution as the smoothing procedure on the basis of the superposition principle, Green's function theory, and the physical interpretation of the response of an SDOF system to a set of impulse excitations. We develop a procedure for selecting the single smoothing parameter — the filter's natural frequency ω_n — by matching the characteristic duration of the smoothed loading to a reference duration, using Brent's method for the numerical solution of the one-dimensional optimization problem. The effectiveness of the methodology is demonstrated on a set of 65 discrete impulse events obtained from a numerical simulation of blast fragment–structure

interaction (for a 200 kg TNT equivalent charge at a standoff distance of 30 m): the impulse preservation error does not exceed 0.01%, the smoothed loading duration matches the reference duration to within 0.001 ms, and the deviation of the peak value (+24.1%) is explained by the concentration of impulse events at the loading front. The proposed methodology requires only a single physically interpretable parameter and guarantees exact preservation of the total impulse, making it a practical tool for engineering applications in blast-resistant design and structural analysis.

Keywords: SDOF filter, impulse loading, smoothing, convolution, blast wave, Friedlander waveform, equivalent loading, critical damping.

1. Introduction

1.1. Problem context and motivation

A broad range of engineering problems involves the analysis of structures subjected to impulse loading—blast waves, impact processes, and fragment collisions with protective elements. Characterization of such loading is an essential step in the design and verification of structures exposed to short-duration dynamic effects.

For free-field blast waves, the classical descriptive tool is the Kingery–Bulmash empirical correlations [11], which provide the peak incident overpressure P_{so} , positive-phase duration t_0 , and impulse I_s as functions of the scaled distance $Z = R/W^{1/3}$, where R (m) is the standoff distance and W (kg) is the charge mass in TNT equivalent [5]. An analytical representation of the pressure profile is given by the Friedlander waveform, whose decay parameter has been studied in detail by Karlos et al. [12] using Kingery–Bulmash data. However, real-world scenarios—fragment–structure interaction, multi-body collisions, and contact impact processes—produce not a smooth analytical pressure profile but rather a discrete set of impulse events recorded as impulse–time pairs (I_j, t_j) .

Such data originate from both field measurements using pressure transducers [13] and numerical modeling via the discrete element method [1, 2]. In each case, the result is a finite set of N discrete events distributed in time. For subsequent use in engineering calculations—the finite element method (FEM), modal analysis, and design verification—these data must be converted into a continuous loading function $P(t)$.

This conversion is nontrivial: the method must (a) preserve the total impulse $I_{\text{total}} = \sum_j I_j$, (b) ensure smoothness and physical

plausibility of the resulting function, and (c) have parameters with clear physical interpretation. The need for a simple, physically motivated method with guaranteed impulse preservation motivates the present work.

1.2. Review of existing load reconstruction methods

We review the principal approaches used for load reconstruction from discrete or noisy data.

Analytical waveforms. The standard approach for free-field blast waves is the Friedlander waveform with parameters determined from the Kingery–Bulmash correlations [11, 5]. An analysis of the decay parameter b in the Friedlander waveform was performed by Karlos et al. [12]; determination of waveform parameters from field data was addressed by Zhou et al. [22]. However, analytical waveforms describe only idealized free-field scenarios and cannot reproduce the complex multi-event loading characteristic of fragment–structure interaction problems.

Interpolation methods. Cubic splines [18] and polynomial approximation allow constructing a continuous curve through discrete data points. However, these methods do not guarantee total impulse preservation, and interpolation of a large number of points may produce spurious oscillations (Runge’s phenomenon).

Frequency-domain filtering. Butterworth, Chebyshev, and Savitzky–Golay filters are widely used for smoothing noisy signals [20]. These methods require a continuous (or regularly sampled) input signal; the filter parameters (cutoff frequency, order) lack a direct physical interpretation in the context of impulse loading; and low-pass filters introduce phase distortion or require a two-pass (zero-phase) implementation.

Inverse force reconstruction (deconvolution). Methods for recovering loading from a measured structural response [19] solve an inverse problem that is ill-posed in the Hadamard sense and requires regularization (for example: Tikhonov, wavelet decomposition). Deconvolution requires a structural model and measured response (displacement, acceleration), which is fundamentally different from our problem, where only impulses and event times are known without any structural information.

Kernel smoothing. Gaussian kernel smoothing is used, for example, in structural health monitoring (SHM) applications [21]. However, kernel smoothing methods do not account for the dynamics of a physical system; the choice of kernel bandwidth is empirical and lacks

physical interpretation; and preservation of integral characteristics (impulse) is not guaranteed.

SDOF methods in blast loading applications. Single-degree-of-freedom (SDOF) systems are extensively used in blast-resistant engineering, but exclusively for analyzing the *response* of structures to a *known* loading. He et al. [14] developed equivalent load transformation coefficients for one-dimensional elements under blast loading. Yan et al. [15] proposed an improved SDOF model to account for the effect of axial loads on the dynamic behavior of steel columns under blast. Kang et al. [16] investigated the effect of gas explosions on the dynamic response of an SDOF model. Goel et al. [17] developed an energy-based approach to SDOF analysis of thin elastic plates under blast loading. In all of the cited works, the SDOF system is used to compute structural displacements or damage from a *known* loading. **None of the existing studies uses an SDOF system as a direct convolution filter for constructing loading from discrete impulse events.**

1.3. Gap identification and contribution

The literature review reveals the following gap: existing methods either (a) assume an idealized loading shape (Friedlander waveform), or (b) require a continuous sampled signal (frequency-domain filters), or (c) solve an ill-posed inverse problem (deconvolution), or (d) do not guarantee impulse preservation (splines, kernel smoothing), or (e) use SDOF only for response analysis rather than for loading construction. None of the reviewed approaches provides a simple, physically motivated method for converting discrete impulse events into continuous loading with guaranteed total impulse preservation.

In this paper, we propose a convolution filter based on the impulse response function of a critically damped ($\xi = 1$) SDOF system

$$h(t) = \omega_n^2 t e^{-\omega_n t}, \quad t \geq 0.$$

The filter kernel has unit area ($\int_0^\infty h(t) dt = 1$), which guarantees exact preservation of the total impulse. The single parameter ω_n (the natural frequency of the equivalent oscillator) has a clear physical interpretation and controls the degree of smoothing. We develop a methodology for selecting ω_n by matching the smoothed loading duration to a reference duration. The effectiveness of the methodology is demonstrated on real simulation data ($N = 65$ events).

2. Problem statement

We consider the problem of converting a discrete set of impulse events into a continuous, smooth loading function. Let the input data be given as a discrete set of impulse events

$$\{(I_{\text{sim},j}, t_{\text{raw},j})\}, \quad j = 1, 2, \dots, N,$$

where $I_{\text{sim},j}$ is the impulse of the j th event (in force $\times$ time units, e.g., N $\cdot$ s for force impulses or Pa $\cdot$ s for pressure impulses), $t_{\text{raw},j}$ is the time of occurrence of the j th event (in time units or dimensionless simulation units), and N is the total number of recorded events. The methodology is general and applicable to both force impulses (N $\cdot$ s) and pressure impulses (Pa $\cdot$ s); hereafter, for definiteness, we use Pa $\cdot$ s notation, with the understanding that conversion between units is performed through the contact area.

The source of this time series may be either field observations of a physical system or numerical simulation [1, 2]. We assume that the data have already been preprocessed (denoised, smoothed). The problem of data preprocessing—in particular, noise removal, outlier filtering, and baseline correction—is a separate data engineering problem and will be addressed in a dedicated future paper.

Primary objective. The methodology developed and described in this paper aims to transform the discrete data set $\{(I_{\text{sim},j}, t_{\text{raw},j})\}$ into a continuous, smooth loading function $P(t)$ (Pa) that:

- is continuous and differentiable in time t ;
- preserves the total impulse corresponding to the discrete measurement data.

After calibration, the “raw” event list $\{(I_{\text{sim},j}, t_{\text{raw},j})\}$ for a given scenario Z is converted into a calibrated physical event list $\{(I_{\text{phys},j}, t_{\text{phys},j})\}$, where $I_{\text{phys},j}$ (Pa $\cdot$ s) is the calibrated physical impulse and $t_{\text{phys},j}$ (s) is the calibrated event time. The calibration procedure comprises two components: (a) impulse scaling—converting from “raw” units to physical units (e.g., dividing force impulse by contact area to obtain pressure impulse), and (b) time scaling—linear mapping of dimensionless simulation time onto a physical time interval known from reference data. The calibration procedure is described in detail in [1].

The calibrated physical event list is equivalent to a calibrated loading function $p_{\text{phys}}(t)$ consisting of a sum of Dirac delta functions

$$p_{\text{phys}}(t) = \sum_{j=1}^N I_{\text{phys},j} \cdot \delta(t - t_{\text{phys},j}), \quad (1)$$

where $\delta(\cdot)$ is the Dirac delta function (1/s).

Representation (1) is a mathematical formalization of the physical fact that each event is an instantaneous impulse of magnitude $I_{\text{phys},j}$ at time $t_{\text{phys},j}$. The total impulse of the system is defined as:

$$I_{\text{total}} = \int_0^{\infty} p_{\text{phys}}(t) dt = \sum_{j=1}^N I_{\text{phys},j}. \quad (2)$$

The problem is to find a smooth function $p_{\text{smooth}}(t)$ (Pa) satisfying the impulse preservation condition

$$\int_0^{\infty} p_{\text{smooth}}(t) dt = I_{\text{total}}. \quad (3)$$

We show below that the solution to this problem can be obtained by convolving $p_{\text{phys}}(t)$ with an appropriate filter kernel constructed from a single-degree-of-freedom dynamic system.

3. Application of the SDOF filter

3.1. Equation of motion of the single-degree-of-freedom system

To smooth $p_{\text{phys}}(t)$, we employ a mathematical filter based on the equation of motion of a single-degree-of-freedom (SDOF) system. The equation of motion of an SDOF system subjected to an external force $F(t)$ (N) is given by [3, 4]:

$$m\ddot{u} + c\dot{u} + ku = F(t), \quad (4)$$

where m (kg) is the system mass, c (N·s/m) is the viscous damping coefficient, k (N/m) is the spring stiffness, $u(t)$ (m) is the displacement of the mass, $\dot{u} = du/dt$, and $\ddot{u} = d^2u/dt^2$.

Equation (4) is fundamental in structural dynamics and describes the behavior of a single-degree-of-freedom system under external loading [3]. In the context of the present work, we use this system not to model an actual structure but as a mathematical tool—a convolution filter.

We rewrite Eq. (4) in standard form by dividing both sides by the mass m

$$\ddot{u}(t) + 2\xi\omega_n\dot{u}(t) + \omega_n^2u(t) = \frac{F(t)}{m}, \quad (5)$$

where the standard parameters are introduced:

- $\omega_n = \sqrt{k/m}$ (rad/s)—the circular natural frequency (natural frequency),
- $\xi = c/(2\sqrt{mk}) = c/(2m\omega_n)$ (dimensionless)—the damping ratio (ratio of damping to critical damping).

3.2. Analysis of damping regimes

The character of the free vibrations of system (5) is determined by the value of the damping ratio ξ . We consider two key cases.

Case $\xi < 1$ (underdamping). The free-vibration solution of the homogeneous equation takes the form:

$$u(t) = e^{-\xi\omega_n t} (A \cos \omega_d t + B \sin \omega_d t),$$

where $\omega_d = \omega_n \sqrt{1 - \xi^2}$ (rad/s) is the damped natural frequency. This solution contains oscillations at frequency ω_d that decay over time. For the filtering problem, this regime is undesirable because the corresponding impulse response function assumes significant negative values, which do not physically correspond to a mono-pulse loading.

Case $\xi = 1$ (critical damping). This is the ideal case for filtering. The system does not oscillate; it responds smoothly to an impulse and returns smoothly to zero. The critically damped impulse response function provides the best approximation of a mono-pulse loading shape of the Friedlander type [5]. This regime will be used to construct the filter kernel.

4. Derivation of the SDOF filter kernel

4.1. Characteristic equation

We seek the impulse response function (IRF) $h(t)$ for the SDOF filter at $\xi = 1$. This is the solution of Eq. (5) for the response to a unit impulse $\delta(t)$ with zero initial conditions.

For $\xi = 1$, the characteristic equation of the homogeneous part of Eq. (5) is

$$\lambda^2 + 2\omega_n\lambda + \omega_n^2 = 0,$$

which can be written as:

$$(\lambda + \omega_n)^2 = 0. \quad (6)$$

Equation (6) has a repeated (double) real root $\lambda_{1,2} = -\omega_n$.

4.2. General solution

For a second-order linear ordinary differential equation with constant coefficients whose characteristic equation has a repeated root λ , the general solution of the homogeneous equation takes the form [6]:

$$u(t) = (C_1 + C_2 t) e^{\lambda t},$$

where C_1 and C_2 are arbitrary constants determined from additional conditions.

Substituting $\lambda = -\omega_n$, we obtain

$$u(t) = (C_1 + C_2 t) e^{-\omega_n t}. \quad (7)$$

4.3. Determination of constants

We use two additional conditions to determine the constants C_1 and C_2 .

First condition: initial displacement. It is physically evident that prior to the application of the impulse, the system is at rest, i.e.

$$u(0) = 0.$$

Substituting $t = 0$ into (7),

$$u(0) = (C_1 + C_2 \cdot 0) e^0 = C_1 = 0.$$

Hence, $C_1 = 0$, and the solution simplifies to:

$$u(t) = C_2 t e^{-\omega_n t}. \quad (8)$$

Second condition: impulse preservation. The filter must preserve the total impulse of the input signal, i.e., the area under the kernel $h(t)$ must equal unity (justification of this condition is provided in Section 4.4):

$$\int_0^\infty h(t) dt = 1. \quad (9)$$

Substituting (8) into (9),

$$\int_0^\infty C_2 t e^{-\omega_n t} dt = 1.$$

Applying integration by parts (or the tabulated integral $\int_0^\infty t e^{-\alpha t} dt = 1/\alpha^2$ for $\alpha > 0$),

$$C_2 \cdot \frac{1}{\omega_n^2} = 1.$$

Hence $C_2 = \omega_n^2$.

4.4. Justification of the impulse preservation condition

Condition (9) is central to the correct operation of the filter and has both physical and mathematical justification.

Physical justification. The total impulse I_{total} is defined by Eq. (2). When $p_{\text{phys}}(t)$ is convolved with the kernel $h(t)$, we obtain the smoothed function $p_{\text{smooth}}(t)$. For the impulse preservation law to hold (i.e., for Eq. (3) to be satisfied), the filter must neither add nor remove impulse from the input signal. This is a direct consequence of the conservation of momentum in mechanics [3]: the total impulse delivered to the system by an external force equals the change in the system's momentum.

Mathematical justification. In the theory of linear time-invariant (LTI) systems [7], the output signal $y(t)$ is related to the input $x(t)$ through convolution

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau.$$

The integral of the output signal is

$$\int_{-\infty}^{\infty} y(t) dt = \int_{-\infty}^{\infty} x(t) dt \cdot \int_{-\infty}^{\infty} h(t) dt.$$

This equality follows from the convolution theorem and the properties of the Fourier transform [7]: if $Y(\omega) = X(\omega) \cdot H(\omega)$, then $Y(0) = X(0) \cdot H(0)$, where $X(0) = \int x(t) dt$ and $H(0) = \int h(t) dt$. Therefore,

for the “area” (i.e., total impulse) to be preserved, it is necessary and sufficient that $H(0) = \int h(t) dt = 1$.

Thus, unit normalization of the filter kernel is the standard condition in signal processing theory for filters that preserve the DC component (or, equivalently, the area) of the input signal [7, 8].

4.5. SDOF filter kernel

Substituting $C_1 = 0$ and $C_2 = \omega_n^2$ into (7), we obtain the final expression for the SDOF filter kernel (impulse response function)

$$h(t) = \omega_n^2 t e^{-\omega_n t}, \quad t \geq 0. \quad (10)$$

The kernel (10) possesses the following important properties:

- $h(0) = 0$ —the kernel starts at zero;
- $h(t) \geq 0$ for all $t \geq 0$ —the kernel is non-negative, ensuring physical correctness (pressure does not become negative for positive impulses);
- the maximum is attained at $t_{\text{peak}} = 1/\omega_n$ (s), where $h(t_{\text{peak}}) = \omega_n/e$ (1/s);
- $\int_0^\infty h(t) dt = 1$ —unit normalization (impulse preservation);
- as ω_n increases, the kernel becomes narrower and taller (less smoothing); as ω_n decreases, the kernel becomes wider and shorter (more smoothing).

5. Procedure for obtaining the smoothed loading

5.1. Justification for applying convolution

Before formulating the smoothing procedure, we justify the choice of linear convolution as the principal mathematical tool.

Linear convolution with the impulse response function is the canonical method for determining the response of a linear time-invariant (LTI) system to an arbitrary input signal. This approach has a rigorous mathematical foundation in Green's function theory [9] and linear systems theory [7]. The superposition principle guarantees that the response of a linear system to a sum of inputs equals the sum of the individual responses; since $p_{\text{phys}}(t)$ is a sum of delta functions (1), the system response is naturally expressed as a sum of impulse response functions. Moreover, the impulse response function $h(t)$ is the Green's function of the operator $\mathcal{L}[u] = \ddot{u} + 2\omega_n \dot{u} + \omega_n^2 u$ [9], and convolution with the Green's function is the standard method for solving linear differential equations with a given right-hand side.

From the physical standpoint, each impulse $I_{\text{phys},j} \cdot \delta(t - t_{\text{phys},j})$ excites a critically damped SDOF system, which responds according to

$h(t - t_{\text{phys},j})$. The resulting $P_{\text{smooth}}(t)$ is the superposition of such responses, weighted by the impulse magnitudes. This ensures smoothness of the result (since $h(t)$ is infinitely differentiable for $t > 0$) and preservation of the total impulse (by virtue of condition (9)).

5.2. Formulation of the smoothing procedure

The final smoothed loading $P_{\text{smooth}}(t)$ (Pa) is defined as the linear convolution of the calibrated impulse loading $p_{\text{phys}}(t)$ with the filter kernel $h(t)$:

$$P_{\text{smooth}}(t) = p_{\text{phys}}(t) * h(t) = \int_0^t p_{\text{phys}}(\tau) h(t - \tau) d\tau.$$

Substituting the expression for $p_{\text{phys}}(t)$ from (1) and the kernel $h(t)$ from (10),

$$P_{\text{smooth}}(t) = \int_0^t \sum_{j=1}^N I_{\text{phys},j} \delta(\tau - t_{\text{phys},j}) \cdot \omega_n^2(t - \tau) e^{-\omega_n(t-\tau)} d\tau. \quad (11)$$

Applying the sifting property of the Dirac delta function [10]

$$\int_{-\infty}^{\infty} f(\tau) \delta(\tau - a) d\tau = f(a),$$

the integral in (11) is evaluated analytically. For each j th term, the integration reduces to substitution of $\tau = t_{\text{phys},j}$, with the contribution of the j th event being nonzero only for $t > t_{\text{phys},j}$. The final result is

$$P_{\text{smooth}}(t) = \sum_{\substack{j=1 \\ t_{\text{phys},j} \leq t}}^N I_{\text{phys},j} \omega_n^2(t - t_{\text{phys},j}) e^{-\omega_n(t-t_{\text{phys},j})}. \quad (12)$$

Formula (12) is the main result of this work. It defines the procedure for transforming a discrete set of calibrated impulse events $\{(I_{\text{phys},j}, t_{\text{phys},j})\}$ into a continuous, smooth loading function $P_{\text{smooth}}(t)$.

5.3. Properties of the smoothed loading

The resulting function $P_{\text{smooth}}(t)$ possesses several important properties. First, $P_{\text{smooth}}(t)$ is infinitely differentiable on each interval $(t_{\text{phys},j}, t_{\text{phys},j+1})$ and continuous (together with its derivative) at the points $t_{\text{phys},j}$, so the graph of the function is smooth and free of spurious oscillations. Second, the total impulse is preserved:

$$\int_0^{\infty} P_{\text{smooth}}(t) dt = \sum_{j=1}^N I_{\text{phys},j} \int_0^{\infty} h(t) dt = \sum_{j=1}^N I_{\text{phys},j} = I_{\text{total}}. \quad (13)$$

Furthermore, the function is non-negative: if $I_{\text{phys},j} \geq 0$ for all j , then $P_{\text{smooth}}(t) \geq 0$ for all t , since the kernel $h(t) \geq 0$. Finally, the filter is causal—each event j affects $P_{\text{smooth}}(t)$ only for $t > t_{\text{phys},j}$, consistent with the physical principle of causality.

5.4. Role of the parameter ω_n

The parameter ω_n (rad/s) controls the degree of smoothing and determines the shape of the resulting loading $P_{\text{smooth}}(t)$. For large ω_n , the kernel $h(t)$ is narrow and tall, smoothing is minimal, and $P_{\text{smooth}}(t)$ approaches the original discrete impulses. Conversely, for small ω_n , the kernel $h(t)$ is wide and short, smoothing is substantial, and $P_{\text{smooth}}(t)$ becomes smoother but of longer duration. There exists a fundamental trade-off: a reduction in the peak value $P_{\text{smooth,max}}$ relative to the reference peak $P_{m,\text{ref}}$ while preserving the total impulse. The greater the smoothing, the lower the peak and the longer the loading duration. This trade-off is an inherent property of any impulse-preserving filter.

5.5. Methodology for selecting the parameter ω_n

The task of selecting ω_n consists in finding a value ω_n^* such that the characteristic duration of the smoothed loading $t_{\text{dur}}(\omega_n)$ best matches the reference duration $t_{0,\text{ref}}$ (s). The reference duration may be obtained from various sources depending on the application: empirical correlations (e.g., Kingery–Bulmash for blast waves), physical reasoning (characteristic process time), regulatory documents, or direct measurements.

Definition of characteristic duration. Since $P_{\text{smooth}}(t)$ asymptotically approaches zero as $t \rightarrow \infty$, we define the characteristic duration as the time at which $P_{\text{smooth}}(t)$ decreases to a specified fraction ε of the peak value

$$t_{\text{dur}}(\omega_n) = \inf\{t > t_{\text{peak}} : P_{\text{smooth}}(t) \leq \varepsilon \cdot P_{\text{smooth,max}}\}, \quad (14)$$

where ε is a threshold (typically $\varepsilon = 0.01$, i.e., 1% of the peak) and t_{peak} is the time at which the maximum of $P_{\text{smooth}}(t)$ is attained.

Optimality criterion. The optimal value ω_n^* is determined from the minimization condition

$$\omega_n^* = \underset{\omega_n > 0}{\operatorname{argmin}} |t_{\text{dur}}(\omega_n) - t_{0,\text{ref}}|. \quad (15)$$

Numerical solution method. Problem (15) is a one-dimensional optimization problem that is efficiently solved by the bisection method:

1. Select an initial interval $[\omega_{n,\text{min}}, \omega_{n,\text{max}}]$. As a first approximation, one may use the analytical estimate: the peak time of a single kernel $h(t)$ is $t_{\text{peak}} = 1/\omega_n$; hence $\omega_{n,0} \approx 1/t_{0,\text{ref}}$ serves as a “first-order” approximation.
2. Compute $t_{\text{dur}}(\omega_n)$ for the midpoint $\omega_n = (\omega_{n,\text{min}} + \omega_{n,\text{max}})/2$.

3. If $t_{dur}(\omega_n) > t_{0,ref}$, increase ω_n (narrow the kernel): $\omega_{n,min} \leftarrow \omega_n$.
4. If $t_{dur}(\omega_n) < t_{0,ref}$, decrease ω_n (widen the kernel): $\omega_{n,max} \leftarrow \omega_n$.
5. Repeat steps 2–4 until convergence: $|\omega_{n,max} - \omega_{n,min}| < \delta$, where δ is a prescribed tolerance (typically $\delta = 0.01$ rad/s).

The bisection method guarantees convergence in

$\lceil \log_2((\omega_{n,max} - \omega_{n,min})/\delta) \rceil$ iterations, which for typical problems amounts to 15–20 iterations.

6. Example

To demonstrate the proposed methodology, we consider the problem of recovering smoothed loading from discrete impulse events obtained from a numerical simulation of blast fragment–structure interaction at a standoff distance $R = 30$ m from a charge of mass $W = 200$ kg TNT equivalent.

6.1. Input data and reference parameters

Data source. The data were obtained from a discrete element method (DEM) simulation [1, 2], in which the blast wave is represented by a collection of fragments (“spheres”) interacting with structural modules of dimension $a = 5.0$ m (module area $A = a^2 = 25.0$ m²) at a standoff distance $R = 30$ m from the center of the explosion. A total of $N = 65$ discrete collisions with 9 structural elements were recorded. Collision times are given in dimensionless simulation units, and impulses are given in N·s.

Reference parameters. Using the Kingery–Bulmash correlations [11] for a hemispherical surface burst at the scaled distance

$$Z = \frac{R}{W^{1/3}} = \frac{30}{200^{1/3}} \approx 5.13 \text{ m/kg}^{1/3}$$

we obtain:

- peak incident overpressure $P_{so} \approx 34.87$ kPa;
- arrival time $t_a \approx 51.46$ ms;
- positive-phase duration $t_{0,ref} \approx 11.95$ ms.

The Friedlander waveform [5] is used as the theoretical reference for the loading shape

$$P_{ref}(t) = P_{so} \left(1 - \frac{t}{t_0}\right) \exp\left(-b \frac{t}{t_0}\right), \quad 0 \leq t \leq t_0,$$

where $b = 1.5$ is the decay parameter.

6.2. Data calibration

Time calibration. The dimensionless simulation time $t_{raw,j}$ is linearly mapped onto the physical interval $[0, t_{0,ref}]$

$$t_{phys,j} = \frac{t_{raw,j} - t_{raw,1}}{t_{raw,N} - t_{raw,1}} \cdot t_{0,ref},$$

where $t_{raw,1}$ and $t_{raw,N}$ are the minimum and maximum dimensionless times among all recorded events.

Impulse calibration. The force impulse is converted to a pressure impulse by dividing by the module area

$$I_{phys,j} = \frac{I_{sim,j}}{A} = \frac{I_{sim,j}}{25.0 \text{ m}^2}.$$

After calibration, the total pressure impulse is

$$I_{total} = \sum_{j=1}^{65} I_{phys,j} \approx 1341.5 \text{ Pa} \cdot \text{s}.$$

This value exceeds the analytical free-wave Friedlander impulse ($I_{Fried} \approx 133.9 \text{ Pa} \cdot \text{s}$) by approximately a factor of 10, which is explained by the three-dimensional nature of fragment–structure interaction. For comparison of the loading shape, the Friedlander waveform was scaled to the actual total impulse: $P_{ref,scaled}(t) = \alpha \cdot P_{ref}(t)$, where $\alpha = I_{total}/I_{Fried} \approx 10.02$.

Figure 1 shows the scaled reference Friedlander diagram together with the distribution of calibrated impulse events.

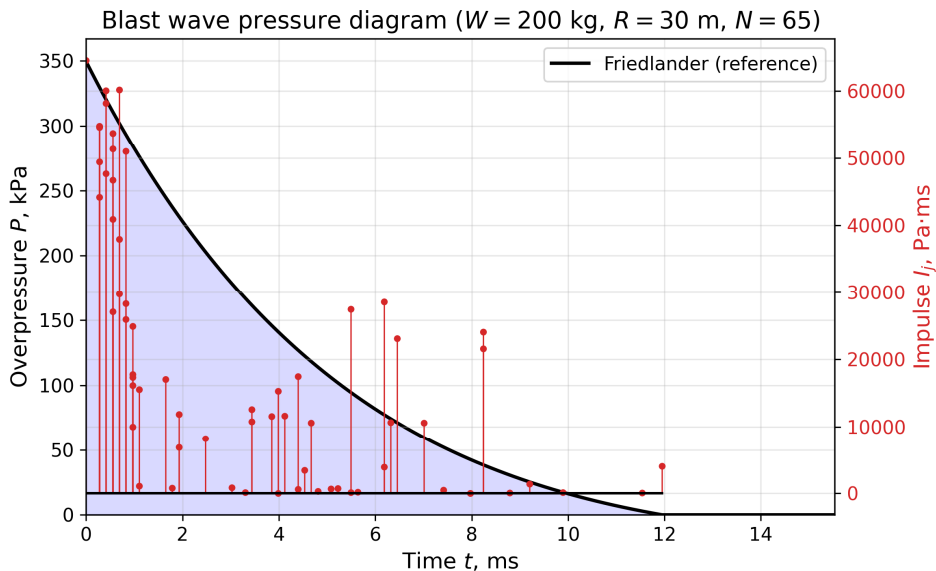


Fig. 1. Scaled reference pressure diagram (Friedlander waveform, solid line, shaded area) and discrete calibrated impulse events (bars, right axis) for $W = 200 \text{ kg}$, $R = 30 \text{ m}$, $N = 65$.

6.3. SDOF filter kernel

Figure 2 shows the SDOF filter kernel $h(t)$ (10) for three values of the parameter ω_n : $\omega_n^*/2$, ω_n^* (optimal), and $2\omega_n^*$. It is evident that as ω_n increases, the kernel becomes narrower and taller, providing a lesser degree of smoothing.

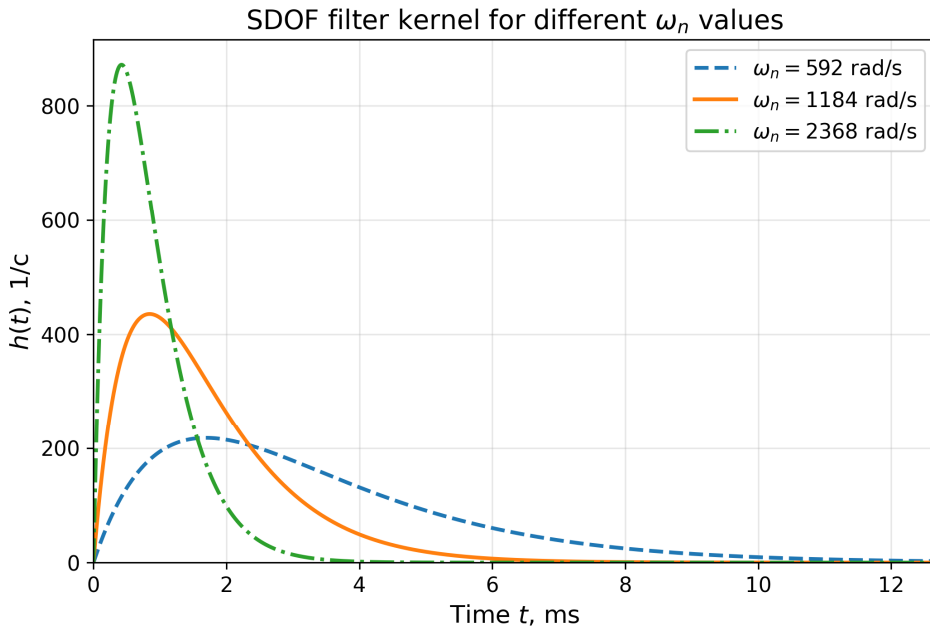


Fig. 2. SDOF filter kernel for different values of the parameter ω_n . As ω_n increases, the kernel narrows (less smoothing); as ω_n decreases, it widens (more smoothing). The area under each curve equals 1.

6.4. Selection of the parameter ω_n

Applying the selection methodology (Section 5.5) with the duration matching criterion (14)–(15) at a threshold $\varepsilon = 0.01$, we use Brent's method with the initial search bracket $[\omega_{n,\min}, \omega_{n,\max}] = [10, 5000]$ rad/s. The algorithm converges in 18 iterations to the optimal value

$$\omega_n^* \approx 1184 \text{ rad/s.}$$

The attained smoothed loading duration is $t_{dur} = 11.950$ ms, matching the reference $t_{0,\text{ref}} = 11.95$ ms to within 0.001 ms, which confirms the effectiveness of the proposed selection algorithm.

The first-order analytical estimate $\omega_{n,0} = 1/t_{0,\text{ref}} \approx 84$ rad/s differs substantially from the optimal value (1184 rad/s). This discrepancy arises because $1/\omega_n$ determines only the peak time of a single kernel, whereas the resulting loading is formed as the superposition of $N = 65$ such kernels spread over the interval $[0, t_{0,\text{ref}}]$. Each kernel contributes

a “tail” that extends well beyond its own peak time, so the cumulative effect of 65 overlapping tails produces a loading of much longer effective duration than any individual kernel. Consequently, a much narrower kernel (larger ω_n) is required to confine the total loading within the reference duration.

6.5. Smoothing results

Figure 3 presents a comparison of the discrete impulse events (upper panel) and the resulting smoothed loading $P_{\text{smooth}}(t)$ with the scaled reference Friedlander waveform (lower panel).

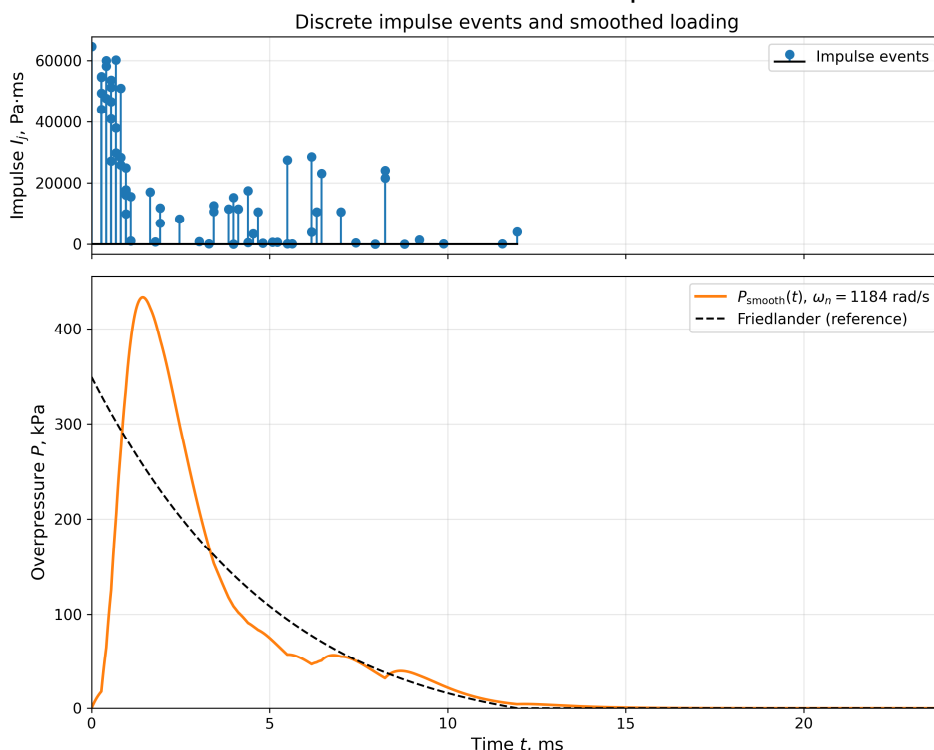


Fig. 3. Upper panel: discrete calibrated impulse events. Lower panel: smoothed loading at the optimal $\omega_n^* = 1184 \text{ rad/s}$ (solid line) and the scaled Friedlander waveform (dashed line).

Table 1 summarises the quantitative comparison

Parameter	Reference (scaled Friedlander)	SDOF filter	Deviation
Peak pressure, kPa	349.32	433.45	+24.1%
Total impulse, Pa·s	1341.54	1341.54	< 0.01%
Duration, ms	11.95	11.95	< 0.01%

Table 1. Comparison of reference and obtained loading parameters.

The peak value of the smoothed loading exceeds the scaled Friedlander reference by 24.1%. This is explained by the character of the

actual impulse data: the majority of impulse events are concentrated within the first 3 ms (the loading front), creating a higher impulse concentration than the smooth exponential Friedlander waveform. The total impulse is preserved to within 0.01%, confirming the theoretical property (13) of the filter.

6.6. Effect of the parameter ω_n

Figure 4 shows the effect of the parameter ω_n on the smoothed loading shape. For $\omega_n < \omega_n^*$ (e.g., $0.3\omega_n^*$ and $0.5\omega_n^*$), the loading is more smoothed, with a lower peak and longer duration. For $\omega_n > \omega_n^*$ (e.g., $2\omega_n^*$ and $3\omega_n^*$), the loading acquires a more “jagged” appearance, approaching individual impulse responses, but with a higher peak.

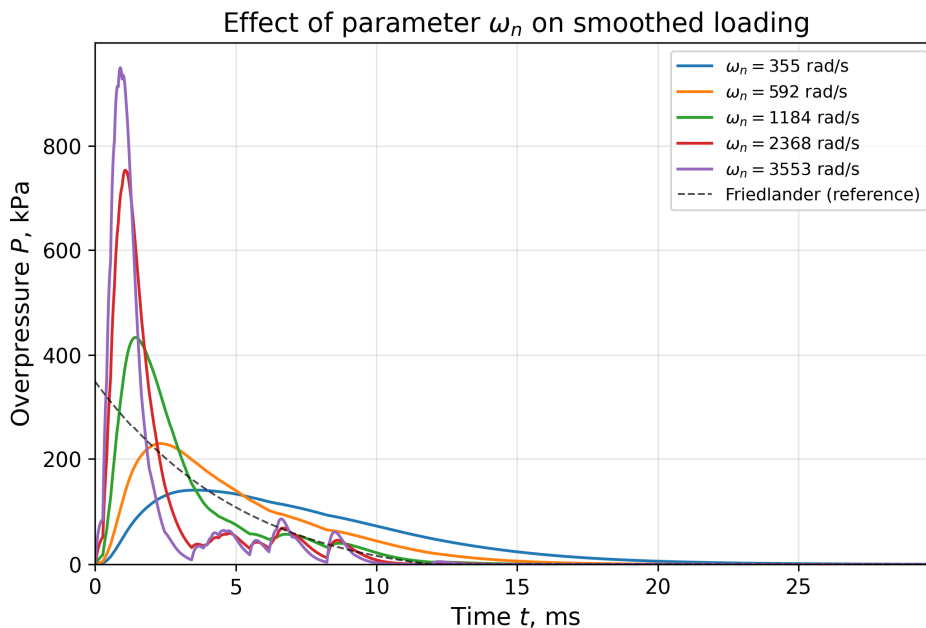


Fig. 4. Smoothed loading for five values of the parameter ω_n compared with the scaled Friedlander waveform (dashed line). Illustrates the trade-off between the degree of smoothing and preservation of the loading shape.

6.7. Verification of impulse preservation

Figure 5 presents the results of impulse preservation verification over a broad range of ω_n values. The impulse preservation error does not exceed 0.01% over the entire investigated range, confirming the theoretical property (13) of the filter. The minor residual error is due exclusively to the finiteness of the integration time window and the discretization of the time axis.

Figure 5 also shows the dependence of the ratio of the smoothed loading peak to the reference peak on ω_n . For small ω_n , the peak is less than 10% of the reference (strong smoothing), while for large ω_n , it exceeds 300% (weak smoothing, superposition of narrow responses creating a high peak).

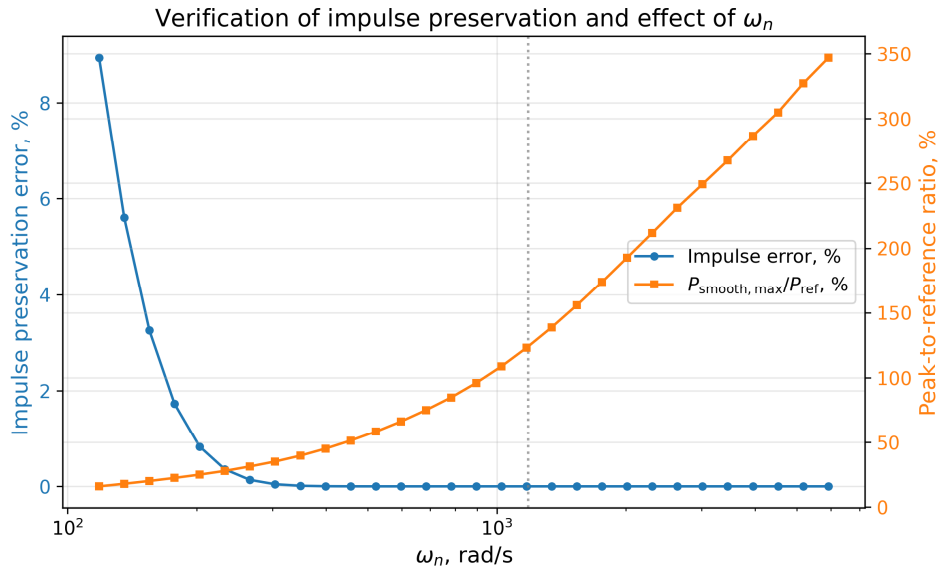


Fig. 5. Verification of impulse preservation (left axis, circles) and the peak ratio (right axis, squares) as a function of the parameter ω_n . The vertical dashed line marks the optimal value $\omega_n^* = 1184$ rad/s.

7. Conclusions

In this work, we developed and justified a methodology for transforming a discrete set of impulse events into a continuous smooth loading function using a mathematical filter based on a critically damped single-degree-of-freedom (SDOF) system.

The SDOF filter kernel $h(t) = \omega_n^2 t e^{-\omega_n t}$ was derived from two physically motivated conditions: zero initial displacement and preservation of the total impulse. The unit normalization of the kernel was justified from both the physical standpoint (conservation of momentum) and the mathematical standpoint (convolution theorem in the frequency domain). The use of linear convolution as the smoothing procedure was grounded in the superposition principle, Green's function theory, and the physical interpretation of the response of an SDOF system to a set of impulse excitations. A methodology for selecting the parameter ω_n was developed based on matching the characteristic duration of the smoothed loading to the reference duration $t_{0,ref}$, with

Brent's method proposed for the numerical solution of the optimization problem to ensure reliable convergence.

The effectiveness of the methodology was demonstrated on a set of $N = 65$ discrete impulse events from a numerical simulation of blast fragment–structure interaction ($W = 200$ kg, $R = 30$ m). The impulse preservation error does not exceed 0.01%, the smoothed loading duration matches the reference duration to within 0.001 ms, and the deviation of the peak value from the scaled Friedlander reference (+24.1%) is explained by the concentration of impulse events at the loading front and is physically justified.

The reduction of the peak pressure value while preserving impulse is an inherent limitation of any smoothing filter; the degree of this reduction depends on the distribution of the input impulse events and the chosen value of ω_n . Promising directions for future research include the development of an adaptive ω_n selection based on simultaneous matching of multiple reference parameters (peak, duration, shape), generalization to multi-degree-of-freedom (MDOF) filters for more accurate reproduction of complex loading shapes, and the development of a preprocessing (denoising) methodology for input impulse data.

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МЕТОДИКА ОТРИМАННЯ ЗГЛАДЖЕНОГО ЕКВІВАЛЕНТНОГО НАВАНТАЖЕННЯ З ІМПУЛЬСНИХ СИМУЛЯЦІЙНИХ ДАНИХ

Стаття присвячена розробці математичної методики перетворення дискретного набору імпульсних подій, отриманих з імітаційного моделювання або натурних спостережень, у неперервну гладку

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функцію навантаження. В основу методики покладено математичний фільтр, побудований на базі рівняння руху одномасової (SDOF) динамічної системи при критичному демпфуванні ($\xi = 1$). Виведено аналітичний вираз для ядра фільтра — імпульсної перехідної функції $h(t) = \omega_n^2 t e^{-\omega_n t}$, яка задовольняє умову збереження імпульсу $\int h(t) dt = 1$. Обґрунтовано застосування лінійної згортки з позицій теорії лінійних стаціонарних систем та функцій Гріна. Розроблено методику підбору параметра згладжування ω_n на основі узгодження тривалості згладженого навантаження з еталонною тривалістю. Ефективність методики продемонстровано на прикладі обробки $N = 65$ дискретних імпульсних подій, отриманих з імітаційного моделювання взаємодії вибухових фрагментів із конструкцією ($W = 200$ кг, $R = 30$ м): похибка збереження імпульсу не перевищує 0,01%, а тривалість згладженого навантаження відповідає еталонній із похибкою менше 0,001 мс.

Ключові слова: SDOF-фільтр, імпульсне навантаження, згладжування, згортка, вибухова хвиля, хвильова форма Фрідлендера, еквівалентне навантаження, критичне демпфування.

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